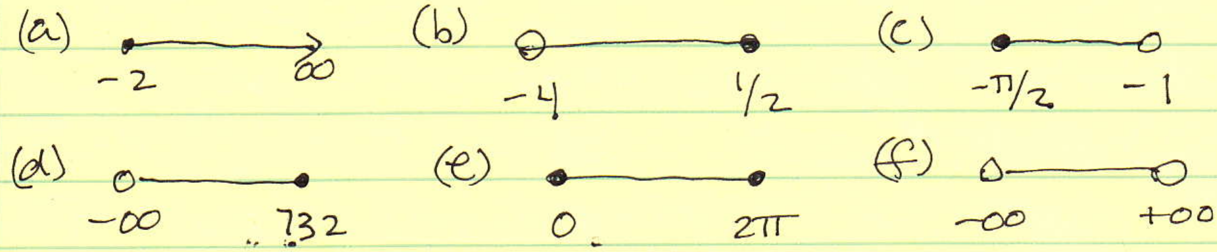
Algebra-Trig Test dueker 2010

1.



2. (a) $\dfrac{(4a^4)^2}{(b^5)^2}\,\dfrac{b^3}{a^7} = \dfrac{16a^8b^3}{b^{10}a^7} = \boxed{\dfrac{16a}{b^7}}$

(b) $\dfrac{1}{t+2} + \dfrac{t}{t-1} - \dfrac{3t}{t^2+t-2} = \dfrac{(t-1)+t(t+2)-3t}{(t+2)(t-1)}$

$t-1+t^2+2t-3t \doteq t^2-1 \Rightarrow \dfrac{(t+1)\cancel{(t-1)}}{(t+2)\cancel{(t-1)}} = \boxed{\dfrac{t+1}{t+2}}$

(c) $\dfrac{1}{y} + \dfrac{y}{y-1} = \dfrac{(y-1)+y^2}{y(y-1)} \Rightarrow$

$\boxed{\dfrac{y^2+y-1}{y-1}}$

$\dfrac{2}{y-1} - \dfrac{1}{y} = \dfrac{2y-(y-1)}{y(y-1)} \Rightarrow$

(d) $\dfrac{3(x-2)^2}{\sqrt[4]{x+5}} + \dfrac{\sqrt[4]{(x+5)^3}\,(x-2)}{1} = \dfrac{3(x-2)^2+(x+5)(x-2)}{(x+5)^{1/4}}$

$3(x^2-4x+4) + x^2+3x-10 = 4x^2-9x+2$

$\boxed{\dfrac{4x^2-9x+2}{(x+5)^{1/4}}}$

3. (a) $1 - 7x \leq 3 + 4x \Rightarrow 11x \geq -2$ $\quad \boxed{x \geq -\frac{2}{11}}$

(b) $|-5x + 3| \leq 4 \Rightarrow -4 \leq -5x + 3 \leq 4$

$\Rightarrow -7 \leq -5x \leq 1$ $\quad \boxed{\frac{7}{5} \geq x \geq -\frac{1}{5}}$

(c) $x^3 - x^2 < 0 \Rightarrow x^2(x-1) < 0$

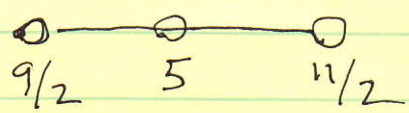
note: $x^2 \geq 0 \Rightarrow x - 1 < 0 \Rightarrow \boxed{x < +1}$

(d) $0 < |x-5| < 1/2 \Rightarrow$ $\quad 0 < x - 5 \quad\quad 0 > x - 5$

$x > 5 \quad\quad x < 5$

$x - 5 < 1/2 \quad\quad x - 5 > -1/2$

$x < 11/2 \quad\quad x > 9/2$



4. skip

5. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad P_1 = \begin{vmatrix} 2 \\ 5 \end{vmatrix} \quad P_2 = \begin{vmatrix} 4 \\ -7 \end{vmatrix}$

$d = \left((4-2)^2 + (-7-5)^2 \right)^{1/2} = \sqrt{4 + 144} = \sqrt{148}$

$d = \sqrt{4 \cdot 37} = \sqrt{4} \cdot \sqrt{37} = \boxed{2\sqrt{37}}$

6. (a) $P_1 = (0,3)$ $P_2 = (2,1)$ $m = \frac{1-3}{2-0} = \frac{-2}{2} = -1$

$$\boxed{y = -x + 3}$$

(b) $P_1 = (-1,-1)$ $P_2 = (1\frac{1}{2}, 3)$ $m = \frac{3+1}{\frac{1}{2}+1} = \frac{4}{1\frac{1}{2}} = \frac{8}{3}$

$$y = mx + b = \frac{8}{3}x + b$$

$$\Rightarrow -1 = -\frac{8}{3} + b \quad b = \frac{5}{3} \Rightarrow \boxed{y = \frac{8}{3}x + \frac{5}{3}}$$

7. A function is a mathematical relation that 'maps' each element of the domain to only one element in the range.

8. The vertical line test states that if a vertical line intersect the ~~function~~ GRAPH more than once, it is not a function.

9. A function f is increasing on an interval, if for any two numbers x_1 and x_2 with $x_1 < x_2$, $f(x_1) < f(x_2)$.

11. $f(x) = 3 - 4^x + x^2 - 5x - 1/3x - \sqrt{x}$

(a) $f(1) = 3 - 4 + 1 - 5 - 1/3 - 1 = \boxed{-6\tfrac{1}{3}}$

(b) $f(0) =$ undefined due to divide by zero.

(c) $f(-1) =$ undefined as $\sqrt{-1}$ is complex number.

(d) $f(a) = 3 - 4^a + a^2 - 5a - 1/3a - \sqrt{a}$

(e) $f(a+h) = 3 - 4^{a+h} + (a+h)^2 - 5(a+h) - 1/3(a+h) - \sqrt{a+h}$

note: to give real result $a+h > 0$ required.

(f) $f(x+h) = 3 - 4^{x+h} + (x+h)^2 - 5(x+h) - 1/3(x+h) - \sqrt{x+h}$

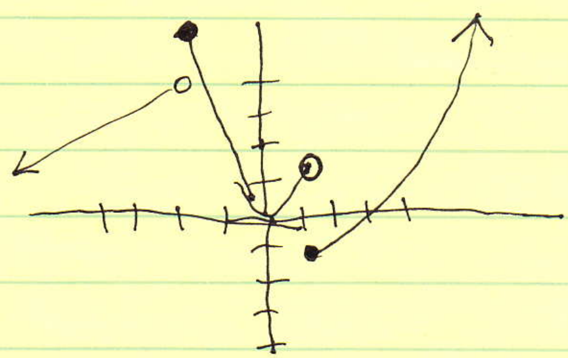
12. $f \circ g(x) = \sqrt{x - 19}$

13. (a) $f \circ g(x) = 1 - x^{-9}$

(b) $h \circ f(x) = \cos(1 - x^9)$

(c) $f \circ g \circ h(x) = 1 - \left(\frac{1}{\cos x}\right)^9 = 1 - \sec^9 x$

14.



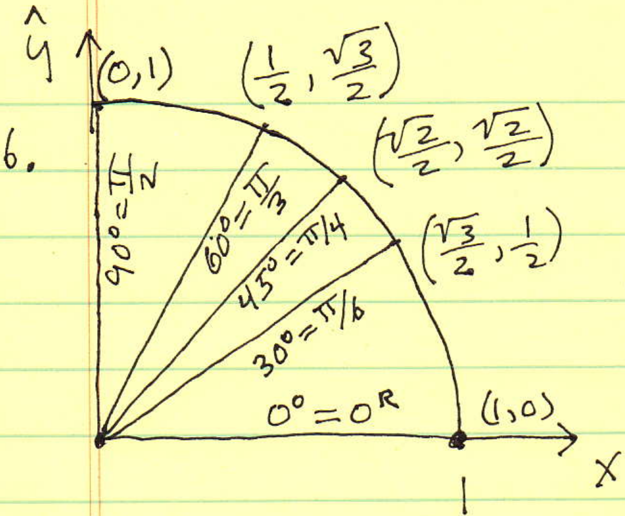
15. (a) $\boxed{0^\circ = 0^R}$ (b) $(3\pi/8)^R / \pi^R = x/180^\circ$ $\quad x = 3/8 \cdot 180$

$x = 3/(\cancel{4} \cdot 2) \cdot (\cancel{4} \cdot 45) = 135/2 = \boxed{67.5^\circ}$

(c) $\dfrac{275^\circ}{180^\circ} = \dfrac{R}{\pi^R} \Rightarrow R = \pi(275/180) = \boxed{4.7997 \text{ radians}}$

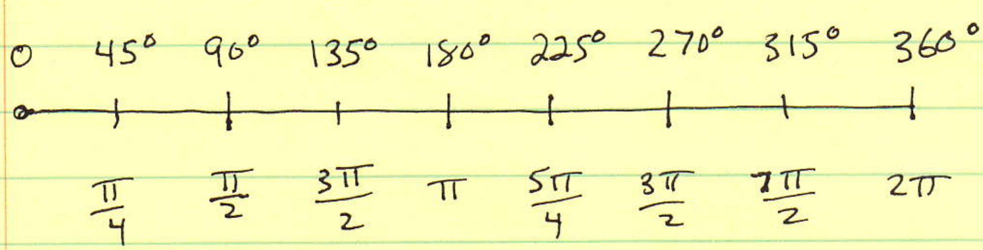
(d) $\dfrac{\pi/2}{\pi} = \dfrac{D}{180} \Rightarrow D = 180 \cdot 1/2 = \boxed{90^\circ}$

16.



$$\frac{n\pi}{6} \quad \frac{n\pi}{4} \quad \frac{n\pi}{3} \quad \frac{n\pi}{2}$$

$$n \cdot 30^\circ \quad n \cdot 45^\circ \quad n \cdot 60^\circ \quad n \cdot 90^\circ$$



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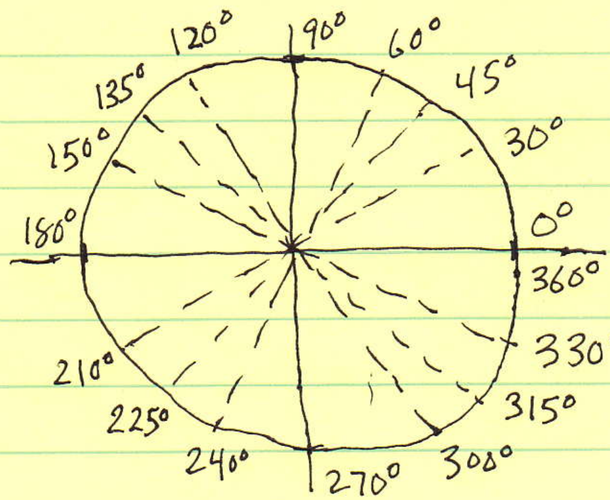

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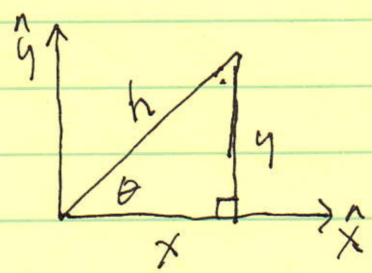
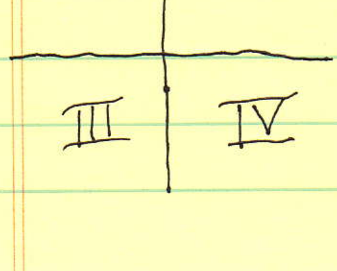


$\sin\theta = y/h \qquad \cos\theta = x/h$

$\tan\theta = y/x$

19.



sin positive quadrant I, II
cos positive quadrant I, IV
tan positive quantrant I, III

20. $\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

21. SOH $\rightarrow \sin\theta = \dfrac{opp}{Hyp}$ CAH $\rightarrow \cos(\theta) = \dfrac{Adj}{Hyp}$

TOA $\rightarrow \tan\theta = opp/Adj$

22.

		$\sin\theta$	$\cos\theta$	$\tan\theta$
$0°$	0	0	1	0
$90°$	$\pi/2$	1	0	∞
$180°$	π	0	-1	0
$270°$	$\frac{3\pi}{2}$	-1	0	∞
$360°$	π	0	1	0

The sign of A, B, C angles depends on quadrant.

(A) $\pi/6,\ 5\pi/6,\ 7\pi/6,\ 11\pi/6$ $\sin\theta = \pm\frac{1}{2}$ $\cos\theta = \pm\frac{\sqrt{3}}{2}$ $\tan\theta = \pm\frac{1}{2}$

(B) $\pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4$ $\sin\theta = \pm 1/\sqrt{2}$ $\cos\theta = \pm 1/\sqrt{2}$ $\tan\theta = \pm 1$

(C) $\pi/3,\ 2\pi/3,\ 4\pi/3,\ 5\pi/3$ $\sin\theta = \pm\sqrt{3}/2$ $\cos\theta = \pm 1/2$ $\tan\theta = \pm\sqrt{3}$

23. $\sin(-7\pi/6) = \sin(2\pi - 7\pi/6) = \sin(5\pi/6) = \boxed{1/2}$

$\cos(-7\pi/6) = \cos(7\pi/6) = \boxed{-\sqrt{3}/2}$

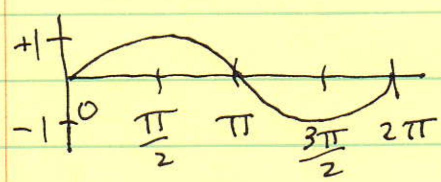
24. $$\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \Rightarrow \boxed{1 + \cot^2\theta = \csc^2\theta}$$

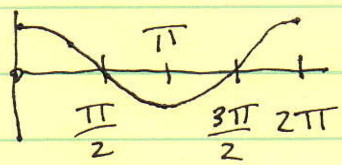
$$\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} \Rightarrow \boxed{\tan^2\theta + 1 = \sec^2\theta}$$

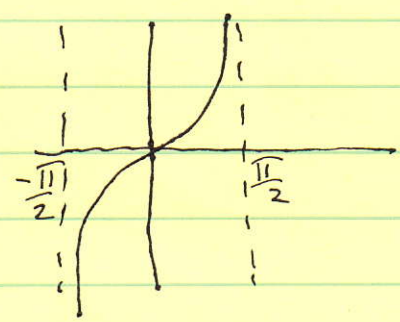
$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

25. $\sin\theta$ is odd function because $f(x) = -f(x)$
$\cos\theta$ is even function because $f(x) = f(-x)$

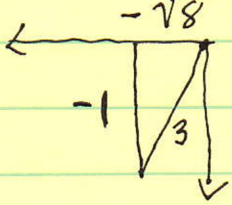
26. $\sin\theta = \sin(\theta + 2\pi n)$ n: integer 2π period.
$\cos\theta = \cos(\theta + 2\pi n)$ n: integer 2π period.
$\tan\theta = \tan(\theta + n\pi)$ n: integer π period.

27. 





28. $\sin\beta = -1/3$ $\pi < x < 3\pi/2$ $\therefore$ in quadrant III

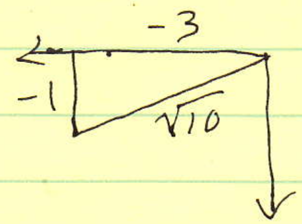


$\cos\beta = -\sqrt{8}/3$ $\tan\beta = 1/\sqrt{8}$

$\csc\beta = -3$ $\sec\beta = -3/\sqrt{8}$

$\cot\beta = \sqrt{8}$

29. $\cot\beta = 3 \quad \pi < x < 2\pi \quad \Rightarrow \tan\beta = 1/3$



$\sin\beta = -1/\sqrt{10} \quad \cos\beta = -3/\sqrt{10}$

$\tan\beta = 1/3 \quad \sec\beta = -\sqrt{10}/3$

$\csc\beta = -\sqrt{10}$

30. (a) $\sin\theta\cot\theta = \dfrac{\sin\theta}{\tan\theta} = \dfrac{\sin\theta}{\sin\theta}\cos\theta = \boxed{\cos\theta}$

(b) $\sec y - \cos y = \dfrac{1}{\cos y} - \cos y = \dfrac{1-\cos^2 y}{\cos y}$

$\dfrac{\sin^2 y}{\cos y} = \dfrac{\sin y}{\cos y}\cdot\sin y = \boxed{\tan y \sin y}$

(c) $\cos[\pi/2 - x] = \cos(\pi/2)\cos x + \sin(\pi/2)\sin(x)$

$= 0\cdot\cos x + 1\cdot\sin x = \boxed{\sin x}$

(d) $\sin(x+y)\sin(x-y) = (\sin x\cos y + \cos x\sin y)(\sin x\cos y - \cos x\sin y)$

$= \sin^2 x\cos^2 y - \cos^2 x\sin^2 y = (1-\cos^2 x)\cos^2 y - \cos^2 x\sin^2 y$

$= \cos^2 y - \cos^2 x\cos^2 y - \cos^2 x\sin^2 y = \cos^2 y - \cos^2 x(\cos^2 y - \sin^2 y)$

$= \cos^2 y - \cos^2 x = (1-\sin^2 y) - (1-\sin^2 x)$

$= -\sin^2 y + \sin^2 x = \boxed{\sin^2 x - \sin^2 y}$

31. $\sin(\pi/3) = x/4$

$\dfrac{\sqrt{3}}{2} = \dfrac{x}{4}$

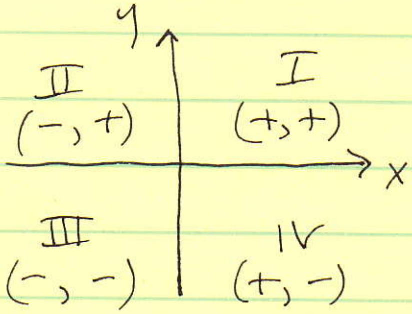
$\boxed{x = 2\sqrt{3}}$

32. (a) $\cos x = 1/2 \Rightarrow x = \pi/3,\ 5\pi/3$

(b) $\sin x = \pm 1/2 \Rightarrow x = \pi/6,\ 5\pi/6,\ 7\pi/6,\ 11\pi/6$

(c) $\sin 2x = \cos x \Rightarrow x = \pi/6,\ 5\pi/6$

(d) $\sin x = \tan x \Rightarrow x = 0,\ 2\pi$



sin I = $^+/_+$ = +
sin II = $^+/_+$ = +
sin III = $^-/_+$ = −
sin IV = $^-/_+$ = −

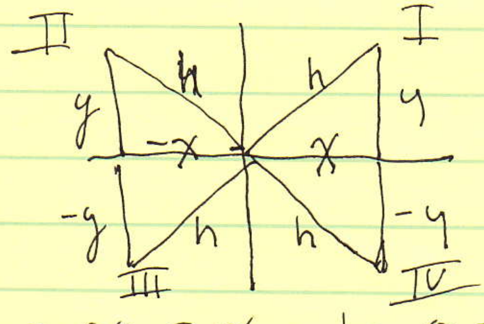
cos I = $^+/_+$ = +
cos II = $^-/_+$ = −
cos III = $^-/_+$ = −
cos IV = $^+/_+$ = +

tan I = $^+/_+$ = +
tan II = $^+/_-$ = −
tan III = $^-/_-$ = +
tan IV = $^-/_+$ = −

φ =	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
φ =	0	$1/2$	$\sqrt{2}/2$	$\frac{\sqrt{3}}{2}$	1
D =	0°	30°	45°	60°	90°
R =	0^R	$\pi/6^R$	$\pi/4^R$	$\pi/3^R$	$\pi/2^R$
x =	1	$\sqrt{3}/2$	$\frac{\sqrt{2}}{2}$	$1/2$	0
y =	0	$1/2$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

	0	30	45	60	90
	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$+\pi/2$	$\pi/2$	$2\pi/3$	$3\pi/4$	$\frac{5\pi}{6}$	π
$+\pi$					
$+3\pi/2$					
$+2\pi$					

$\sin\theta = y/R$ $\sec\theta = R/x$
$\cos\theta = x/R$ $\csc\theta = R/y$
$\tan\theta = y/x$ $\cot\theta = x/y$



R = 1 $\sin\theta = y$ $\cos\theta = x$ $\tan\theta = y/x$
$\sec\theta = 1/x$ $\csc\theta = 1/y$ $\cot\theta = x/y$